Modeling Analysis of Tensile Tests of Bundled Filaments with a Bimodal Weibull Survival Function

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Modeling Analysis of Tensile Tests of Bundled Filaments with a Bimodal Weibull Survival Function

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ABSTRACT: A convolution model applied to single mode Weibull fiber bundle experiments was extended to predict the response of bimodal model bundles. Published data for bimodal SiC fibers predicted the sensitivity of bundle test as a function of gauge length. A bundle test could rapidly determine the modality of a bundle from experiments at two gauge lengths. Next, the method checked the applied strain rate bimodality of Kevlar. The analysis indicates that a bundle test cannot determine the parameters of a model that predicts variation in apparent modulus or strain to failure since slack in the bundle generates these effects.

KEY WORDS: weibull, filament, bundle, fracture, convolution.

INTRODUCTION

FILAMENTS USED AS the reinforcement phase of a high-strength composite material are typically brittle. They exhibit a variation in their strain to failure that requires statistical modeling for the successful use of the filaments in structural applications. Tests of single filaments provide the mean failure strain in as few as ten experiments, however, extracting the shape of the survival function requires testing hundreds of single-fiber samples. This number of tests can require weeks to months of careful sample preparation and analysis. In addition, multiple failure modes are found only by repeating the experiments over a range of conditions such as different gauge lengths and strain rates [1,2]. The long lead-time of the testing means that researchers and manufacturers cannot fully benefit from the data. For example, materials engineers investigating a new filament coating process could use the change in the failure response to indicate rapidly whether the process improves or degrades the filaments. Also, on-line process and quality control

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and could use changes in the failure data to signal problems in a value-added process, e.g., prepregging.

The most straightforward, albeit tedious, method of obtaining values of m and ε_0 collects data from separate tensile tests of a large number of filaments. Separating the filaments from the bundle may introduce damage that shifts the parameters to values that do not represent the response of pristine filaments [1].

For these reasons, testing the bundle of filaments as a whole is an attractive goal for many researchers [3–6]. However, testing the entire bundle introduces other effects [4,7–9]. Breaking filaments may interact with neighboring fibers and cause their premature failure. What is needed is a test method that tests filaments as they exist within the bundle, but with greater independence of filaments from their failing neighbors. Recently, a bundle analysis method proposed for extracting Weibull parameters from tests of filament bundles was successfully applied in experiments for a single mode failure model [6,10]. The Weibull shape and scale parameters extracted from bundles containing 1000 filaments agreed with the results obtained by other researchers who used the single filament tests. Introducing slack into the bundle made the failure more independent and shifted the Weibull shape parameter so that it agreed with the previous literature. The objective of this work is to extend the analysis to bimodal Weibull systems in order to assess the utility of the procedure in reducing the number of experiments needed to extract the failure statistics.

This paper presents three tasks. The first task is extending the convolution analysis to a bimodal Weibull failure model. The second task is using the extended model to predict the tensile response of filament bundles. Predicting the response demonstrates the potential value of the analysis; the results indicate whether the method will provide a significant improvement over the single filament tests. That is, will the predicted sensitivity of the method allow extraction of the bimodal response? The experimental results of Zok et al. provide a rational set ofWeibull parameters for the analysis [1].

Finally, the method will check the results presented by Wang and Xia [2]. Their tests of Kevlar 49 "superbundles" containing 18,000 filaments indicated that the fiber is bimodal with respect to the applied strain rate. Here the data are re-examined using the principles learned from bundle mechanics and experiments [5,6]. This will contrast the two methods and to show that filaments tested in a bundle must be analyzed carefully in order to determine the effect of slack on the extracted parameters.

THEORY

This development of the single mode and bimodal Weibull equation follows the presentation of Wagner [11]. The survival probability, S, of a filament is a function of the volume of material under test, V and the mean volume V_0 associated with the critical flaw. The relationship of survival to volume is

$$S(V) = \exp[W_0^{-1}] \quad 0 \le S(V) \le 1.0 \tag{1}$$

Here the volumes are nondimensional values. For a cylindrical filament the nondimensional volume of the sample under test and the critical flaw volume are expressed by the relations:

$$V = \frac{\pi R^2 L}{\pi R^2 L_0} \quad \text{and} \quad V_0^{-1} = \frac{\pi R^2 L_0}{\pi R^2 L'} \tag{2}$$

where R is the radius of the filament, L is the gauge length of the filament, L' is the length associated with the volume of the critical flaw in the cylindrical filament and L_0 is a reference length chosen for convenience. If the applied strain generates flaws according to a power-law relation, the critical flaw volume becomes this relation:

$$V_0^{-1} = \left(\frac{\varepsilon}{\varepsilon_0}\right)^m \tag{3}$$

where ε is the applied strain, m is the shape parameter, and ε_0 is the scale parameter. Finally, combining Equations (1)–(3) generates the familiar two-parameter Weibull cumulative distribution function:

$$S(\varepsilon) = \exp\left[-\left(\frac{L}{L_0}\right)\left(\frac{\varepsilon}{\varepsilon_0}\right)^m\right] \quad 0 \leq S(\varepsilon) \leq 1.0 \tag{4}$$

The probability that a filament will survive the application of strain varies from a value of 1.0 (filament intact), which occurs at zero strain, to 0.0, which occurs at infinite strain. With some filaments the survival does not scale with length and the shape and scale parameters only apply to the specific length tested, this is the case with Kevlar filaments [11–13]. In this case, the ratio of L/L_0 is sometimes replaced with the value of 1. When applied, L_0 is typically taken as 1 mm although some researchers use 1 m [1]. The shape parameter and the scale parameter control the fit of the equation to a set of data. The set (m, ε_0) defines the failure mode.

When several types of flaws occur in a sample of filaments, each flaw may have its own mode. That is, the set (m_i, ε_i) exists for each flaw type if the flaws cause independent, i.e., noncollaborative, failure events in the fiber. A plot of $\ln(\varepsilon)$ versus $\ln(-\ln(S))$ can make the modes apparent although a linear plot of S versus ε resembles the single mode response. This is a Weibull plot; it shows each m as the slope of the plotted function. Transitions from one linear slope of m to another indicate the presence of another mode.

If the flaws act independently, Equation (4) readily models the system when extra terms appear within the exponential function. The bimodal survival function for independently acting flaws is this equation [1,2]:

$$S(\varepsilon) = \exp\left[-\left(\frac{L}{L_0}\right)\left(\frac{\varepsilon}{\varepsilon_1}\right)^{m_1} - \left(\frac{L}{L_0}\right)\left(\frac{\varepsilon}{\varepsilon_2}\right)^{m2}\right] \tag{5}$$

where the pairs of parameters (m_1, ε_1) and (m_2, ε_2) describe the survival response of each mode. By combining this relation with the convolution relation of Phoenix, a versatile relation for the analysis of bundle experiments results.

Convolution of Survival Function and Filament Length Distribution

Phoenix derived a general load/strain relation for filament bundles in tension [5]. The filament length distribution and the survival function generate the resulting load, $P(\varepsilon)$, in this convolution relation:

$$P(\varepsilon) = AE_f \int_{-\infty}^{\varepsilon} f(\varepsilon - e)g(e)\, de \tag{6}$$

where f is the product of the applied strain and the appropriate survival function, that is $\varepsilon_S(\varepsilon)$, and g is the filament length density function. The filament cross sectional area, A, and the filament modulus, E_f, are typically taken as constant values. The advantage of this relation is that $S(\varepsilon)$ and $g(\varepsilon)$ are not restricted to any specific formula. Any appropriate filament length distribution and survival function combine to predict the load/strain response of the bundle. Conversely, from a measured load/strain response, either the filament length distribution or the survival function can be extracted if the other function is sufficiently well defined [6,10]. However, other factors make bundle tests impractical. This model assumes that the filaments do not interact during failure and they typically have substantial interactions during the fracture process [4]. The typical filament distribution is an experiment in the Dirac function [5,6]. In a Dirac bundle, all filaments have the same length, i.e., zero slack, and they begin to strain at the same time. However, since all filaments strain from the same initial time, filament failures are concentrated in a small range of strain. This range starts just before the peak load and reaches the strain at which the load returns to zero following the fracture of all filaments. With concentrated failures the interactions are high and catastrophic failure is typical.

The convolution relation can aid testing by allowing the extraction of data for tows with slack filament-length distribution [6]. Adding slack to the bundles reduced the interactions and produced Weibull data that compared well with the results of single filament tests. The failures occurred over the applied strain history and the breaks occur with greater independence [6,10]. Small variations in filament length create large effects in the load/strain response which the deconvolution analysis removes [6,10]. The fiber data presented in the next two sections demonstrate different approaches to determining bimodal parameters. For SiC fiber, single filaments extracted from a composite proved that the bimodal survival behavior resulted from damage introduced during the consolidation process [1].

Filaments with Bimodal Weibull Response

Zok et al. tested single SiC filaments at gauge lengths from 5 to 256 mm [1]. They determined that the bimodal response was a result of processing the filaments into a composite since the pristine filament survives tensile testing with a single Weibull mode. Across the fiber lengths tested, the best parameters for predicting the survival were $m_1 = 18$, $\varepsilon_1 = 0.0089$ and $m_2 = 4.3$, $\varepsilon_2 = 0.0036$. Figure 1 shows three survival curves calculated from the data. The lines show the single mode Weibull functions for each mode (m_1, ε_1) independently as well as the bimodal response (heavy line). In this figure, the bimodal transition is obvious since the survival changes from the mode with the

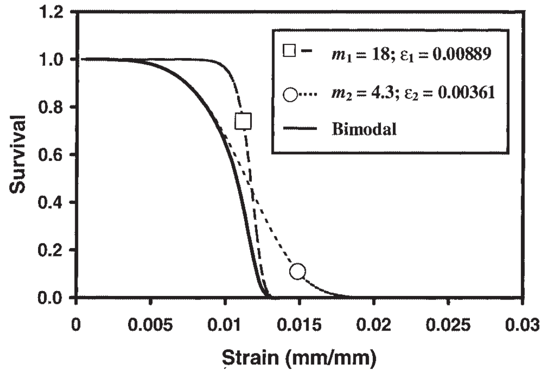

Figure 1. *Survival functions for SiC fiber [1]. Single symbols are used to differentiate the model curves. With a filament length of 5 mm the change in response from one survival curve to the other shows that the SiC is bimodal.*

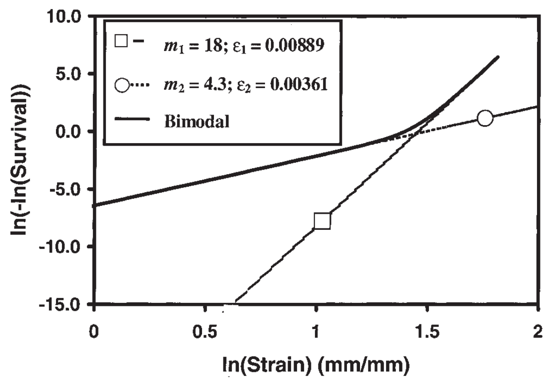

Figure 2. *The SiC fiber data from Figure 1 on a Weibull plot. Symbols differentiate the model curves. The axes emphasize the transition from one shape parameter to the other as strain increases.*

smaller shape parameter at low strain to the mode with the larger shape parameter as strain increases. Figure 2 shows the same model curves on Weibull axes. This figure clearly demonstrates the change from a shape factor of 4.3 to one of 18.0. The model starts with a slope of 4.3, passes through a knee region and then attains a constant slope of 18.0.

Zok et al.'s data set is a good basis for predicting the load/strain response of a bundle of SiC filaments because the Weibull parameters come from interaction-free single filament tests. The Kevlar data discussed next however come from bundle tests and must be carefully reviewed with respect to filament effects.

Filament Bundles with Bimodal Weibull Response

Wang and Xia tested "superbundles" of Kevlar filaments over a range of strain rates that spanned seven orders of magnitude [2]. Eighteen bundles of Kevlar 29 mounted in a special fixture formed the superbundles. Figures 3 and 4 show results on Weibull and linear scaled plots. The extracted parameters are summarized in Table 1. They concluded

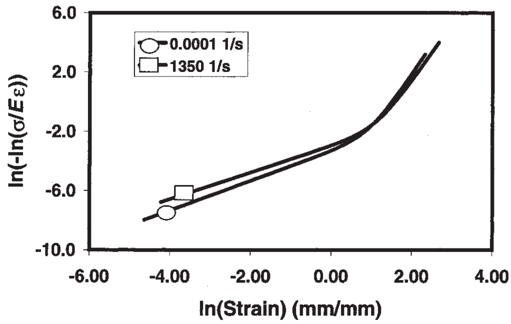

Figure 3. Weibull plot of Kevlar survival data obtained by experiments with superbundles of 18,000 filaments [2]. Wang and Xia's model predicted constant values for the shape parameter. The extracted data show the Kevlar to be bimodal as a function of the strain rate.

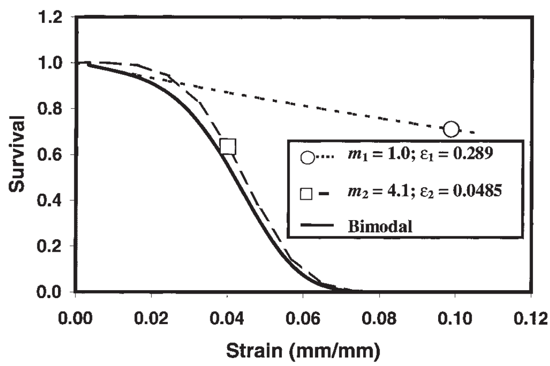

Figure 4. Survival functions for Kevlar fiber on a linear plot [2]. Functions shown are for a strain rate of 10^{-4} s^{-1}.

Table 1. Bimodal Weibull parameters for Kevlar-29 superbundles tested at various strain rates [2]. The shape parameters vary about a mean value and the scale parameters increase with strain rate.

	First Set		Second Set	
Strain Rate	m_1	ε_{01}	m_2	ε_{02}
0.0001/s	1.00	0.0289	4.1	0.0485
140/s	1.10	0.0328	4.5	0.0591
440/s	1.10	0.0335	4.2	0.0634
1350/s	0.9	0.0361	3.9	0.0689
Mean	1.0	0.0341	4.2	0.0638
Standard Deviation	±10%	±5.1%	±7.1%	±7.7%

that the strain rate had an effect that, like the length effect of the SiC fiber, required a bimodal survival model.

In deriving a model that would incorporate rate effects, Wang and Xia define the dynamic fiber modulus E^d and strain at peak stress as functions of a normalized strain rate through these relations [2]:

$$\frac{E^d}{E^{dref}} = 1 + h\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right)$$
$$\frac{\varepsilon_f^d}{\varepsilon_f^{dref}} = 1 + \lambda\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{ref}}\right) \quad (7)$$

The superscript d indicates the dynamic, i.e., high strain rate, quality of the variable and the *ref* values are the properties at a reference strain rate selected by the analyst. The functions h and λ are appropriate models of the strain dependent response. If either of the properties is not rate dependent, the relevant function (h or λ) equals zero. By merging these dynamic modulus and strain relations with the bimodal Weibull survival function, Wang and Xia found that the Weibull shape parameters m were independent of strain rate while the scale parameters were functions of h and λ and would therefore be rate dependent if either Young's modulus or failure strain were affected by strain rate. From the discussion of bundle experiments in this and previous publications, this type of formulation is problematic for a bundle experiment [5,6,10]. Slack affects the apparent modulus and the strain to failure strongly. The superbundle experiment only obtains correct functions for h or λ when slack is not present within the bundle. The analysis below will investigate the effect of this on the extraction of strain rate parameters from bundle experiments.

Analysis

In the next sections, the convolution relation allows an exploration of the effects of strain rate and filament length distribution on the load during a tensile extension of bundles. These calculations show the sensitivity of the analysis to changes that occur due to the bimodal nature of the survival function and due to variation in filament length. The published data applied here represent bimodal failure for two classes: length dependent modality and strain-rate dependent modality. Also, note that during analysis of the Kevlar experimental data, the ratio of L/L_0 does not appear within formula since previous studies showed that the Weibull relation does not scale with sample length for that fiber [14].

Filaments with Bimodal Failure as a Function of their Length

The SiC fibers described above have a bimodal survival that is a function of the sample gauge length. The convolution relation allows us to determine rapidly the bimodal response and to reduce filament interactions that affect the Weibull parameters. The procedure is to run a bundle test at gauge length L_1, extract the apparent Weibull parameters and predict the response at length L_2. Experiments at L_2 would confirm the behavior. As a demonstration, the Weibull parameters for SiC provide a means of predicting the load data expected from a series of experiments. First, the survival curves for 100-filament bundles were calculated at bundle lengths of 5 and 100 mm. Figure 5 shows these bimodal survival curves. Second, the survival curve for the 5-mm bundle was

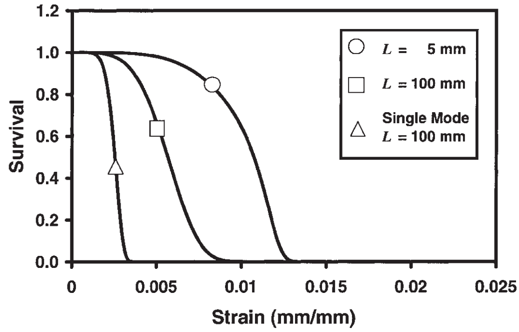

Figure 5. *Predicted survival functions for SiC fiber at 5 and 100 mm gauge lengths as a bimodal and a single mode system without slack in the bundle. Single symbols are used to differentiate the model curves. When apparent two-parameter Weibull values are fit to the 5 mm test (circle) and projected to a 100 mm length, the survival function (triangle) shows a weaker function than should be found by experiment (square).*

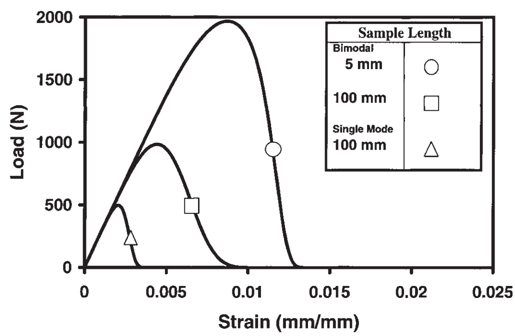

Figure 6. *Predicted load/strain curves for the survival functions in Figure 5. Single symbols differentiate the model curves. When apparent two-parameter Weibull values are fit to the 5 mm test (circle) and projected to a 100 mm length, the load (triangle) should reach one-half the load that should be found by experiment (square) if the material fails with a bimodal response.*

fit by apparent Weibull parameters for a single mode system using Equation (4). Finally, the apparent Weibull survival at 100 mm was calculated as shown in Figure 5. It changed significantly from the response of the bimodal model. This difference should be obvious in the test data.

Figure 6 shows effect of these survival curves on the load/strain data. If the first bundle experiment is run with a gauge length of 5 mm, that is, the highest curve in the figure, a single mode Weibull fit of the curve would predict the result of a bundle test at 100 mm length to be the lowest load/strain curve on the chart. However, if the sample were bimodal in the manner of the SiC filaments, the actual load/strain curve would be the middle result, which has a peak load that is twice as high. Therefore, the test would be a strong indicator of the bimodal effect. Figure 7 shows the impact of slack on the test results as described by Phoenix [5]. The filaments in the 5 mm bundle are uniformly distributed from a short length of 4.98 mm to a largest length of 5.02 mm so that the average filament length is 5.0 mm ± 0.32%. The filaments in the 100 mm bundle vary uniformly from a short filament of 99.7 mm to a maximum length of 100.3 mm so that the average filament length is 100.0 mm ± 0.32%. The modulus of each experiment rises to a

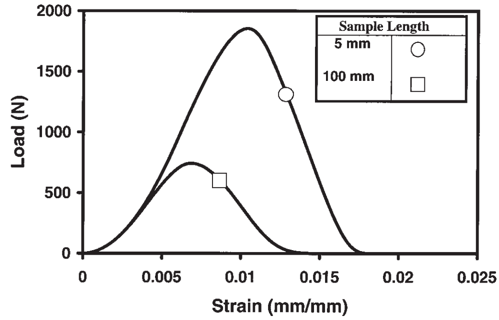

Figure 7. *The predicted effect of slack on a bundle test of SiC filaments. When the filament length varies uniformly by ±0.32% of the sample length, the load strain curves are predicted to exhibit modulus growth at the start of the experiment and the range of strain over which filaments fail increases.*

maximum value before the filaments begin to fail. The modulus has an apparent reduction since the number of filaments in tension does not equal the full set of filaments in the sample. In addition, the fractures spread over a greater range of strain. This application of slack was shown to decrease filament applications and make bundle testing possible for single mode Weibull data [10]. These results show that (1) the effect of variations in the survival function is an easily measured difference in the load/strain response and (2) spreading the filament size over a range of lengths may spread the failures over a greater span of strain and thereby reduce filament interactions. In this manner, bundle experiments might provide a rapid determination of modality in materials that exhibit a length dependent bimodal behavior.

Filaments with Bimodal Failure as a Function of Strain Rate

The Wang and Xia work represents a new capability where high strain rates and superbundles were first applied to extraction of Weibull parameters. Because of this novelty, we must carefully review the data. First, we can compare the low strain rate data to previous studies. Several researches have experimented with bundles and single filaments of Kevlar at strain rates near 0.0001 1/s [9,10,14]. None of those studies found Kevlar to be a bimodal system with one of the shape factors being near a value of 1.0. If the bimodal results are correct,, this prior body of work should note that and it does not. Of course, single filament testing might miss the factor if the total number of filaments tested is less than 100. In that case, the single mode Weibull equation could fit the data without finding the bimodal effect [1]. However, the prior bundle experiments did not indicate this property either [10].

Second, the Kevlar superbundle data shown in Table 1 and Figures 3 and 4 show that the rate dependence, if it exists, is small. While the strain rate changes by seven orders of magnitude, the rate dependent scale parameters increase by only 24 and 42% for ε_{01} and ε_{02} respectively. Since two test machines were used to collect the data, the machine variability alone could account for much of that change. Indeed, the Weibull plot of Figure 3 looks comparable to typical experimental errors presented in other Weibull plots with various fibers [1,14]. Since Wang and Xia did not indicate the expected level of experimental variation, we must look for it in their data. Their model indicates that the shape parameters should be constant for a material. The variation in shape parameter, if the model is correct, provides a measure of the experimental error for the bundle tests since this parameter is rate independent in their model. Using the data in Table 1, the mean and standard deviation were calculated for the experiments conducted at the three highest strain rates. These experiments occurred on the same high rate test machine and using those points will remove the test machine variability that would be present if all of the data were averaged. Table 1 shows that the variation in *m* is as large or larger than the variation in scale parameter, which should be a function of the applied strain rate. Although the values of scale parameter are monotonically increasing with the increase in strain rate, the amount of increase is not larger than expected from experimental variation.

Finally, their data show some interesting differences from other experiments in literature. One noticeable difference is the reported Young's modulus of Kevlar. The modulus of the superbundles only reaches Kevlar-29's typical value of 125 GPa when tested at the highest strain rate. The bundles, if slack free, should exhibit the typical modulus of 125 GPa at the slowest strain rate, which is a typical test condition, and it should change from that value with the increase in strain rate. In addition, prior research has shown significant filament interactions during bundle tests, except in the case of bundles containing slack [10]. Given these factors and the analysis in this work, using bundles to determine an effect that creates changes in modulus and strain to failure is impractical since any slack in the filaments creates the same effects.

Wang and Xia published one chart that showed the raw data obtained with the new test method. Figure 8 shows this raw data plot for the $1350\,s^{-1}$ test compared with the load/strain curve calculated from their bimodal parameters. The shape of the load/strain raw curve has the character of the slack bundle prediction in Figure 7. The raw data extending into negative strain show that the initial portion of the data were truncated before the Weibull parameters were extracted. Perhaps this buildup in modulus was attributed to the compliance of the test machine. If the bundle contained slack filaments, the experiment would have begun at about -0.02 strain in Figure 8. The effect of the slack would be to create apparent changes in modulus and strain to failure.

To explore this possibility, consider an 18 tow superbundle with uniformly distributed slack. The filament lengths vary uniformly from the shortest, which is 7.8 mm long to the longest, which is 8.2 mm in length. This sample would provide an average bundle length of 8.0 mm $\pm 1.4\%$. Using these model parameters–125 GPa modulus, single mode Weibull

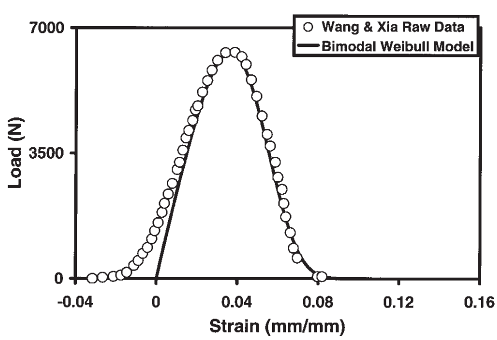

Figure 8. *Kevlar superbundle raw load/strain data compared with the prediction of the bimodal model. The raw data was truncated at the peak modulus [2]. When compared with the prediction in Figure 7, the raw data indicate that slack was present in the bundle.*

parameters of $m_{02} = 4.1$ and $\varepsilon_{02} = 0.0485$, we can generate load/strain predictions for both taut and slack bundles. Figure 9 shows the results and the effect of this small degree of slack is obvious. The horizontal bar that intersects the load/strain curve for the slack bundle indicates the point of maximum modulus for that response and this maximum modulus is 92.8 MPa. The apparent modulus of the bundle with slack is less that the actual modulus by 26%. Continuing with the analysis, the slack curve is now truncated at the point of maximum modulus and shifted to lower strains under this rule – a cubic spline fit of the origin and the curve region following the maximum modulus point must produce an initial modulus that is 125 MPa. This is an arbitrary shift rule. It is plausible, if one does not understand the mechanics of filament bundles to propose this or some other rule as a means of "correcting" the data. This is a common practice with monolithic samples tested under ASTM methods since it removes test machine compliance from the data. However, bundles may contain slack and another method must be applied to characterize and eliminate the compliance. Wang and Xia did not describe the method by which they determined the zero strain level in their experiments. The rule stated here is a plausible means of accomplishing the same result – shifting the load/strain data to eliminate the modulus growth in the raw data. When the shifted load/strain data, also

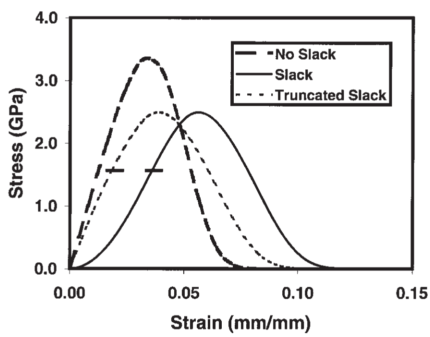

Figure 9. *Predictions of single mode stress strain data for a bundle with and without slack. If the slack bundle data is erroneously truncated and shifted towards the origin as shown, the initial data along the stress–strain curve – under the horizontal bar – are the result of an arbitrary curve fit.*

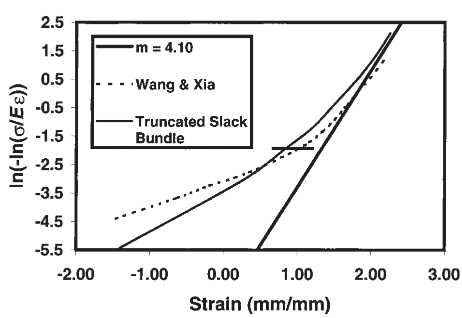

Figure 10. *Weibull scale plot of the bundle predictions from Figure 9. An arbitrary curve fit to the data generated an apparent bimodal effect. The single mode behavior shown by the straight line was turned into the curve solid line response because the mechanics of the tensile response of bundles was not used to extract the Weibull data through the convolution relation.*

shown in Figure 9, is plotted on Weibull paper the result is shown in Figure 10. This analysis started with a single mode material with a shape factor of 4.1. The truncation and shifting of the slack bundle data, in conjunction with a curve fit rule to zero strain impressed a bimodal appearance on the load/strain data. In the case of the arbitrary, yet plausible, rule used here, the apparent bimodal Weibull parameters are $m_1 = 1.4$, $m_2 = 3.9$, $\varepsilon_1 = 0.128$, $\varepsilon_2 = 0.0614$. These parameters are comparable to the Wang and Xia results in Table 1. When analyzed in this way, a single mode Weibull response took on the appearance on bimodal behavior. The horizontal bar in Figure 10 shows the truncation point for this analysis and for Wang and Xia's results. The new mode is almost entirely within the region of the curve fit, which is greatly variable due to the arbitrary nature of such a data shift.

CONCLUSIONS

The convolution analysis could reduce the number of experiments needed to determine the failure parameters of a filament by reducing interactions between filaments through the introduction of slack. The prediction of load/strain behavior developed here shows that the bundle test is highly sensitive to the factor that controls the combined failure response, e.g., sample length. Using three replicate bundles at two test conditions would produce the failure data and modality check in as few as six experiments.

However, the mechanics of bundles in tension must be recognized and understood in order to provide an appropriate test. Bundles always introduce apparent changes in modulus and the range of strain over which filaments fail. Models that expect effects of this type should not use bundled samples for an experiment. The variation due to the bundle mechanics may be greater than the effect sought. At the very least, separating the two causes from the combined effect would be troublesome at best.

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